Preface

This book gives a presentation of some current topics in theory of generalized inverses. As this theory has long been a subject of study of various authors, many of the problems and questions have been resolved. However, some have only been either partially solved or remain still open to this day. It was our goal to use this book to give a review of these efforts as well as to offer the reader possible directions for further study in this area of mathematics as well as hints at possible applications in different types of problems.

This book starts with definitions of various types of generalized inverses and listing the many sorts of applications of them to different branches of both mathematics, but also to some other scientific disciplines, which is aimed at providing motivation behind the study of this topic in general.

Chapter 2 gives an exposition of the so-called reverse order law problem, which is originally posed by Greville as early as in 1960, who first considered it in the case of the Moore–Penrose inverse of two matrices. This was followed by further research on this subject branching in several directions: products with more than two matrices were considered; different classes of generalized inverses were studied; different settings were considered (operator algebras, C^*-algebras, rings, etc.). We discuss the reverse order law for K-inverses, when $K \in \{\{1\},\{1,2\},\{1,3\},\{1,2,3\},\{1,3,4\}\}$ in different settings and present all recently published results on this subject as well as some examples and open problems.

In the next chapter, we first consider problems on completions of operator matrices and then proceed to present a particular approach to solving the problem on the reverse order law for $\{1\}$-generalized inverses of operators acting on separable Hilbert spaces which involves some of the previous research on completions of operator matrices to left and right invertibility. Although the reverse order law problem for $\{1\}$-generalized inverses of matrices was completely resolved by 1998, the corresponding problem for the operators on separable Hilbert spaces was only solved in 2015. So, we thus demonstrate usability of results on completions of operator matrices by showing how they can be applied to one of the topics in generalized inverses of operators that has seen a great interest over the years. Also, in Chap. 3, we consider the problem of existence of Drazin invertible completions of an upper triangular operator matrix and that of the invertibility of a linear combination of operators on Hilbert spaces.

In Chap. 4, we shift our attention from the problem of invertibility a linear combination of operators to that of some different types of generalized invertibility. Special emphasis is put on Drazin and generalized Drazin invertibility of linear combinations of idempotents, commutators, and anticommutators in Banach algebras. Also, some related results are presented on the Moore–Penrose inverse of a linear combination of commuting generalized and hypergeneralized projectors for which certain formulae are considered.

The problem of finding representations of the Drazin inverse of a 2×2 block matrix is of great significance primarily due to its applications in solving systems of linear differential equations, linear difference equations, and perturbation theory of the Drazin inverse. It was posed by S. Campbell in 1983, and it is still unsolved. In Chap. 5, we present all the partial results on this subject that have been obtained so far as well as the different methods and approaches used in obtaining them.

In the last chapter, we present some additive results for the Drazin inverse. Although it was already even in 1958 that Drazin pointed out that computing the Drazin inverse of a sum of two elements in a ring was not likely to be easy, this problem remains open to this day even for matrices. It is precisely on this problem when considered in rings of matrices and Banach algebras that we shall focus our attention here; i.e., under various conditions, we will compute $(P+Q)^{\mathrm{D}}$ as a function of P, Q, P^{D}, and Q^{D}.

This book thus, as readers will surely see for themselves, only tackles some of the current problems of the theory of generalized inverses, but the topics that have been selected have also been thoroughly covered and a systematic presentation given of relevant results obtained so far as well as of possible directions in further research. We should mention that this book has come out as a result of a long and successful collaboration between the authors. Also we were inspired by the work of many colleagues as well as coauthors, some of the joint results with which appear in this book, to whom we are thankful for the experience of working with. Finally, we would like to thank professors Eric King-wah Chu from Monash University and Vladimir Pavlović from the Faculty of Science and Mathematics, University of Niš, who read this book carefully and provided feedback during the writing process.

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